Lecture Topics

In this lecture, we will learn about momentum, Newton's Laws, and inertial and non-inertial reference frames.

Momentum

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We see that the momentum:

- is a vector,
- is in the same direction as the velocity $\vec{v}$, and is proportional to it,
- and is also proportional to the mass m.

Newton's Laws of Motion

We can now state Newton's three Laws of Motion:

First Law If the net force acting on a body is zero, its momentum will not change.

Second Law The rate of change of momentum of a body is given by the net force acting on it.

Third Law For every action, there is an equal and opposite reaction.

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and *usually* the mass m of a body does not change, this implies that the body's velocity $\vec{v}$ will not change.

This leads to another statement of Newton's First Law:

First Law A body at rest will continue to be in a state of rest; a body in a state of uniform linear motion will continue in that state - if the net force acting on that body is zero.

To put this in more mathematical language, we say that if

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 + \ldots = \sum_{i=1,2,3,\ldots} \vec{F}_i = 0,$$

where all these forces are acting on the same body, then:

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When the body is not moving, this figuring out of the forces acting on the body, and how they all balance each other and cancel out, is called **Statics**. This is the science that underlies most of Civil Engineering.

Newton's First Law

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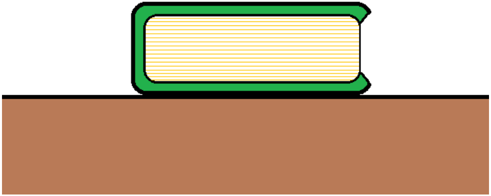

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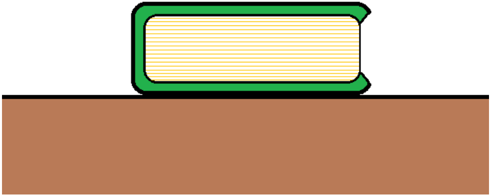
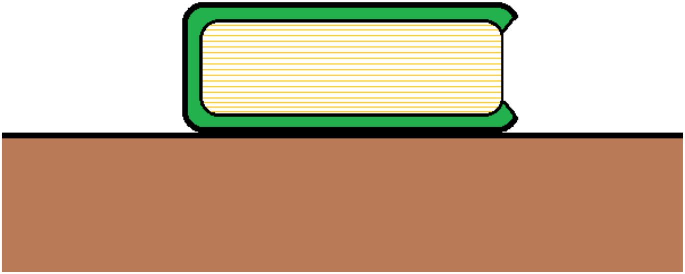


Since its momentum is not changing (it's always zero), the net force acting on the book must be zero.

Newton's First Law

What are the forces acting on the book?



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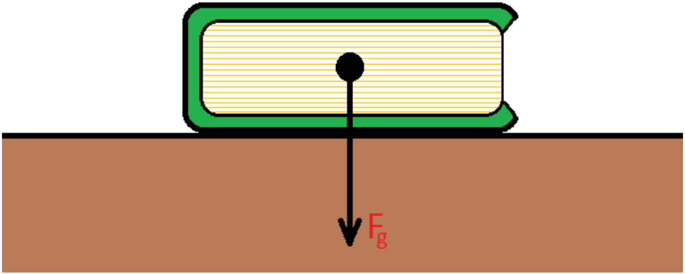

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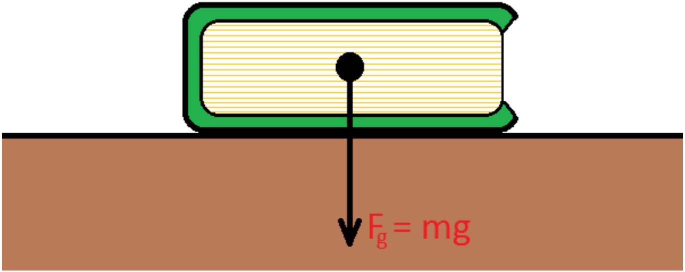

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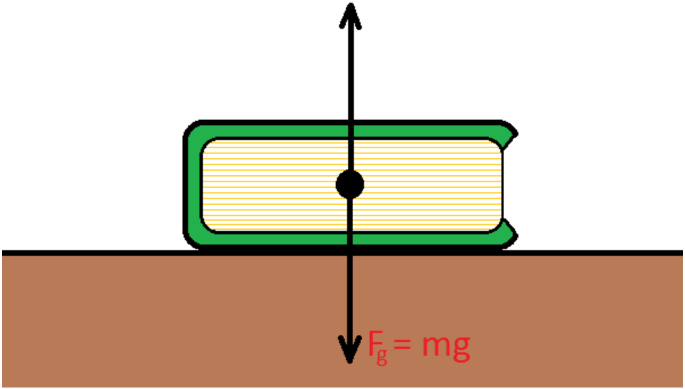

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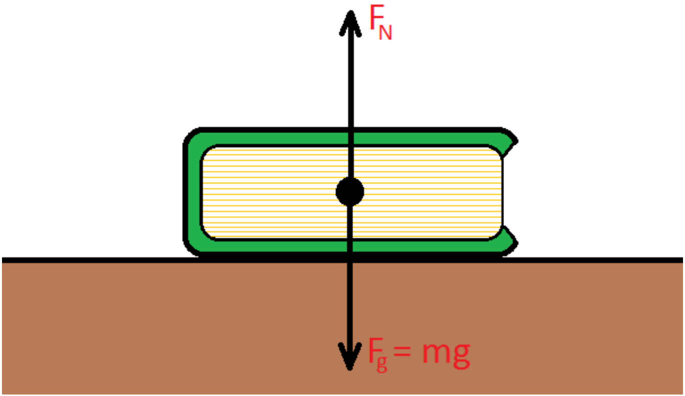

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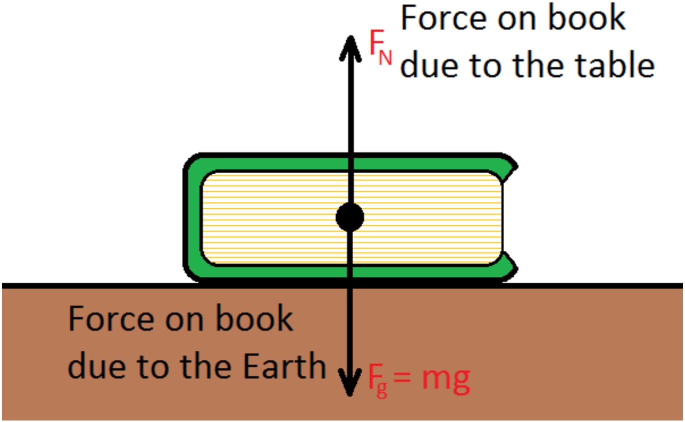

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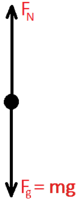

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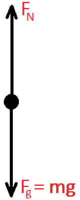
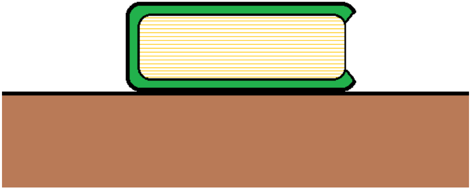


We call this kind of diagram a Free Body Diagram (FBD).

Applying Newton's First Law

When we wish to apply Newton's First Law to a situation:



We look at the physical situation.

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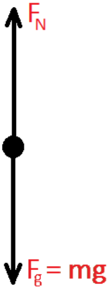

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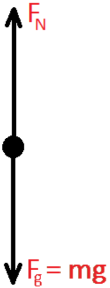

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And then we write the equation

$$\sum_{i=1,2,\ldots} \vec{F}_i = 0$$

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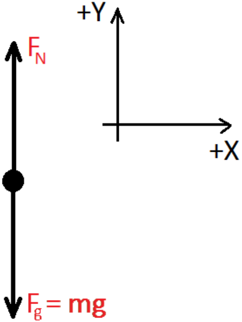

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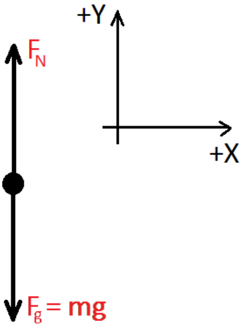

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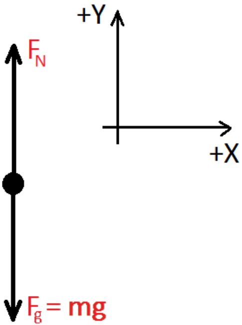

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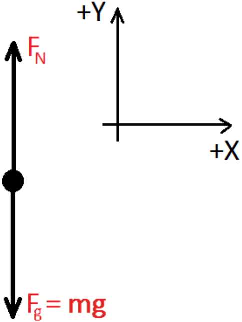
$$\vec{F}_g \cdot \hat{\imath} + \vec{F}_N \cdot \hat{\imath} = \vec{0} \cdot \hat{\imath}$$

$$0 + 0 = 0$$

Applying Newton's First Law



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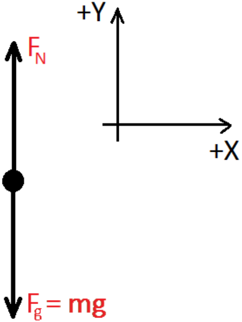

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Note that now in the second of these equations, I am not dealing with vectors, any more, but the magnitudes of vectors, which are scalars. This equation further simplifies to:

$$-F_g + F_N = 0$$
$$-mg + N = 0$$
$$\Rightarrow N = mg$$

We usually use N for the magnitude of the normal force. And we have found that *in this situation*, the magnitude of the normal force is equal to the weight mg of the book.

Newton's Second Law

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Thus we have the famous equation

$$\vec{F} = m\vec{a}$$

and the units of $\vec{F}$ are newtons (N), where $1\,\mathrm{N} = 1\frac{\mathrm{kg\,m}}{\mathrm{s}^2}$.

$$\sum_{i=1,2,3,\ldots} \vec{F}_i = m\vec{a}$$

Variable Mass and Newton's Second Law

$$\sum_{i=1,2,3,...} \vec{F}_i = m\vec{a}$$

It is important to remember the assumption we have made: that m is a constant with respect to time. If the system we are considering does *not* have a constant mass - for example, some exceptions are:

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- a rocket; a rocket burns and expels a large amount of its weight as gas trhough its thrusters. There is a significant loss in weight as the rocket flight progresses

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- a rocket;
- a conveyor belt; if we are, for example, continuously dumping sand or coffee beans on to a conveyor belt, then the mass it is carrying changes

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- a rocket;
- a conveyor belt;
- a hailstone in the process of forming; as a hailstone forms, it acquires layer upon layer of ice as it moves up and down through the cloud. It gains weight while it stays in the cloud

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- a rocket;
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—then in all these cases we must return to the original form of Newton's second law:

$$\sum_{i=1,2,3,\ldots} \vec{F}_i = \frac{\mathrm{d}\vec{p}}{\mathrm{d}t}$$

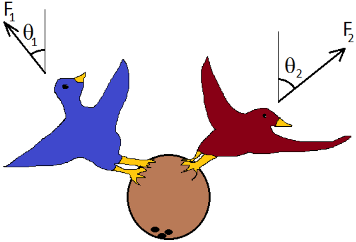
But we will leave these 'variable mass' cases till later.

Problem

Let's consider another problem and work it out on the blackboard

Problem

Two swallows, one European and one African, attempt to lift a coconut of mass $m = 1\,\text{kg}$. The European swallow exerts a force of magnitude $F_1 = 14\,\text{N}$ at an angle of $\theta_1 = 20^o$ from the vertical. The African swallow exerts a force of magnitude $F_2 = 30\,\text{N}$ at an angle of $\theta_2 = 35^o$ from the vertical. In unit vector notation, what is the net acceleration of the coconut? If the coconut starts at a height $y = 0\,\text{m}$ at $t = 0\,\text{s}$, at what time does it reach a height of $y = 20\,\text{m}$?



Not to scale

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- time
- position—the gravitational force between two bodies depends on the distance between them. We have:

  $$F = G\frac{m_1 m_2}{r^2}.$$

  You can see that as r increases, F decreases.

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But not all forces are constant. Some change with

- time
- position
- velocity—the magnitude D of the drag force exerted on a body moving through a fluid can be modelled by

$$D = \frac{1}{2} C \rho A v^2 = B v^2$$

The faster the body moves through the fluid (which could be air, or water, for example), the greater the drag force.

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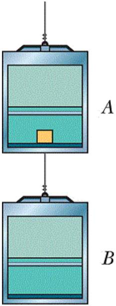
But not all forces are constant. Some change with

- time
- position
- velocity

There are many different kinds of forces, and they can depend on various things. We usually experimentally determine a simple formula which can be used to model their behaviour, and then develop a theory to explain it.

Problem

In the figure, elevator cabs A and B are connected by a short cable and can be pulled upward or lowered by the cable above cab A. Cab A has mass 1700 kg; cab B has mass 1300 kg. A 12.0 kg box of catnip lies on the floor of cab A. The tension in the cable connecting the cabs is 1.91×10^4 N. What is the magnitude of the normal force on the box from the floor?



Reference Frames

So far, I've been ignoring a certain detail. We have seen the first two of Newton's Laws of Motion, but we haven't clarified: are they always true? Are they true for all observers?

Reference Frames

It turns out that Newton's laws are only true for observers in an inertial reference frame.

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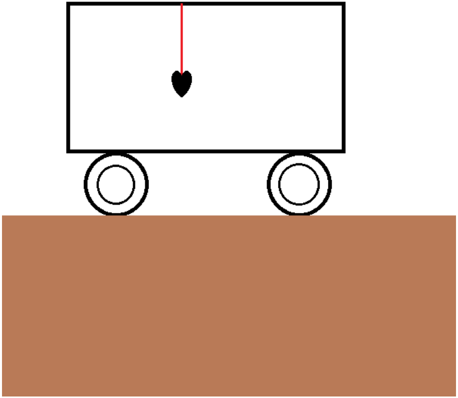
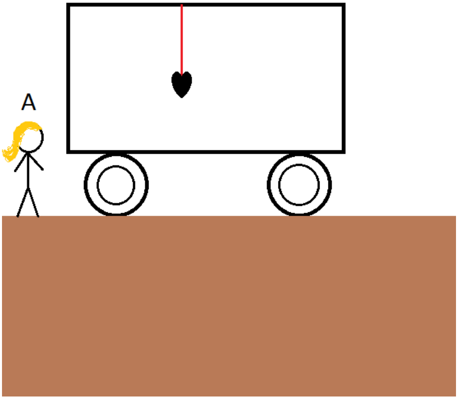
An inertial reference frame is one in which Newton's Laws hold. That is, no matter what experiments the observer performs, everything he sees can be explained as a result of Newton's Laws.

All right, that seems a little circular. The simplest way to get a feeling for this is to look at a non-inertial reference frame. Let us consider a railcar with a mass hanging from a string inside it.



Inertial and Non-inertial Reference Frames

We consider an observer A on the ground outside the railcar, and look at the Free Body Diagram she draws for the mass.



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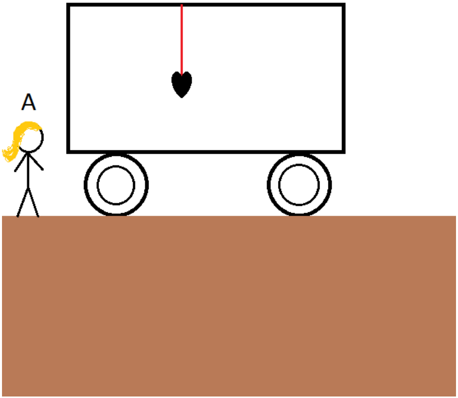
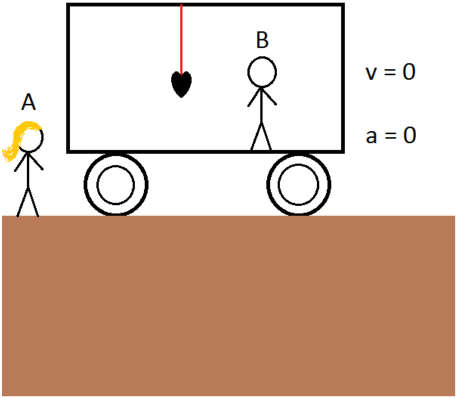
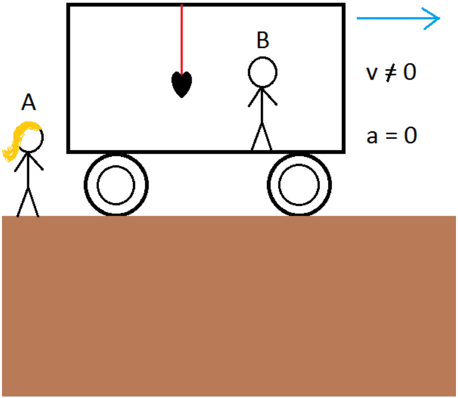


We now consider a second observer B inside the railcar. The railcar is at rest, so the two reference frames are not moving with respect to each other, so they should agree on all the observations they make.



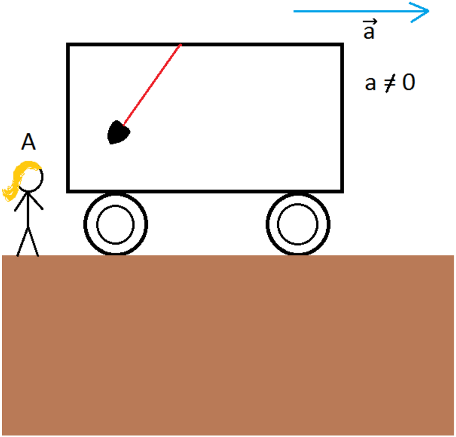
Inertial and Non-inertial Reference Frames

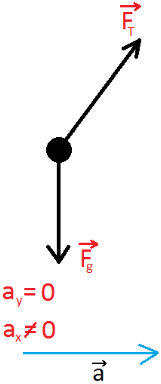
What if the railcar is moving with a constant velocity? Does the Free Body Diagram that B draws change? Do A and B agree on the forces exerted on the mass?



Inertial and Non-inertial Reference Frames

What happens if the railcar is accelerating? Let us look at the Free Body Diagram that A draws.





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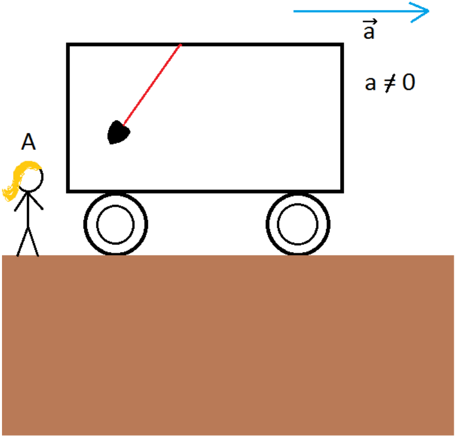
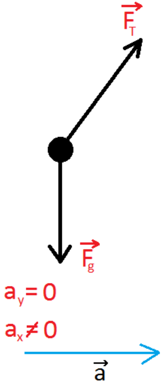
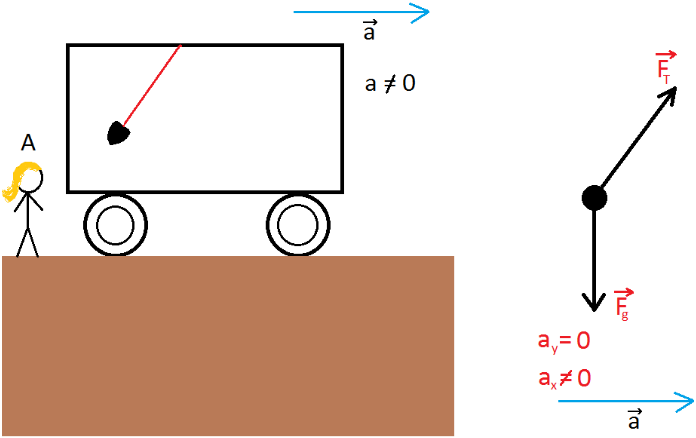




Inertial and Non-inertial Reference Frames

But what about observer B? In his reference frame, *he is stationary*! What does his Free Body Diagram look like?



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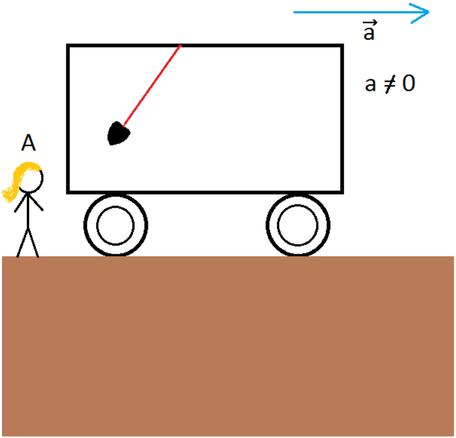
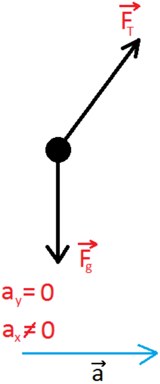
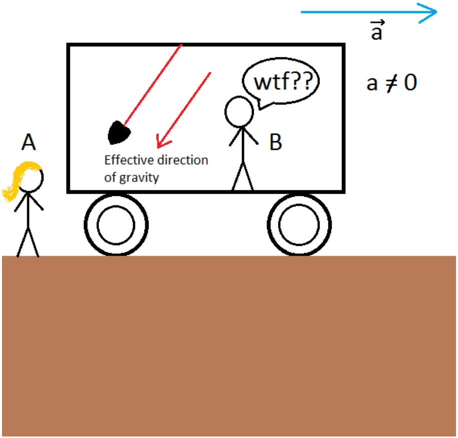




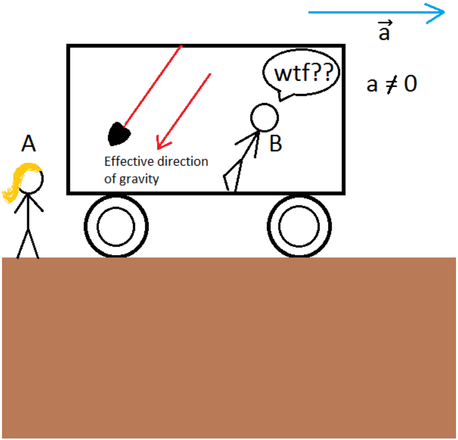
Inertial and Non-inertial Reference Frames

We see that an observer B cannot explain what is happening to the mass with just Newton's Laws. He will have to postulate some new laws of Physics.



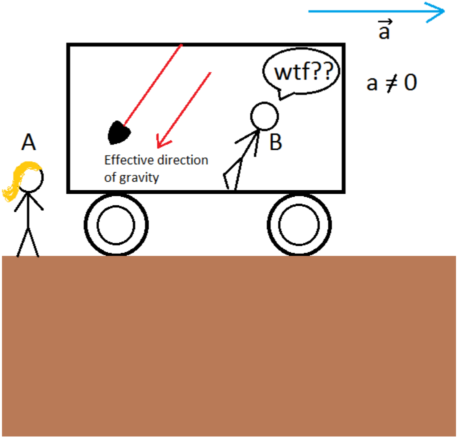
Inertial and Non-inertial Reference Frames

One clue that B will have that something is wrong is that these new forces that he will have to invent, affect everything he sees, in almost the same way. In fact, he will think gravity is acting at a new weird direction.



Inertial and Non-inertial Reference Frames

So, we see here that A is an observer in an inertial frame of reference, and B, when accelerating, is in a noninertial frame of reference.



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So B will see forces acting on P that he cannot explain by Newton's Laws. And these forces will crop up in every Free Body Diagram he draws, in every experiment he does. And this should be a clue to him that he, B, is in a non-inertial frame.

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Newton's Third Law

Third Law For every action, there is an equal and opposite reaction.

If object 1 exerts a force $\vec{F}_{21}$ on object 2, then object 2 exerts and equal (in magnitude) and opposite (in direction) force $\vec{F}_{12}$ on object 1. Mathematically,

$$\vec{F}_{21} = -\vec{F}_{12}$$

So forces always come in pairs. However, it is important to note that these forces act on different objects! $\vec{F}_{21}$ acts on object 2, and $\vec{F}_{12}$ acts on object 1.

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Let's return to the book on the table.

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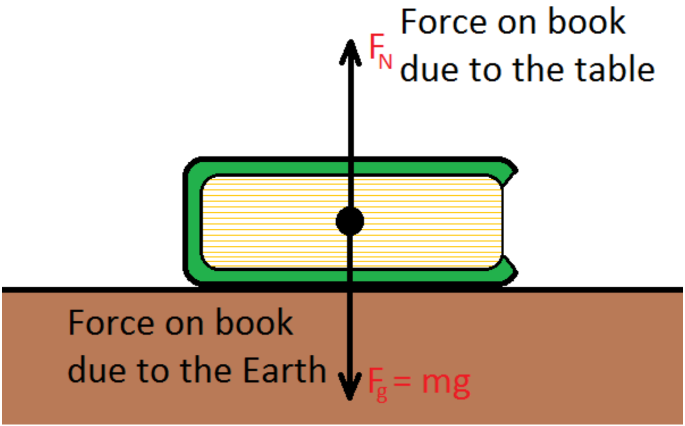

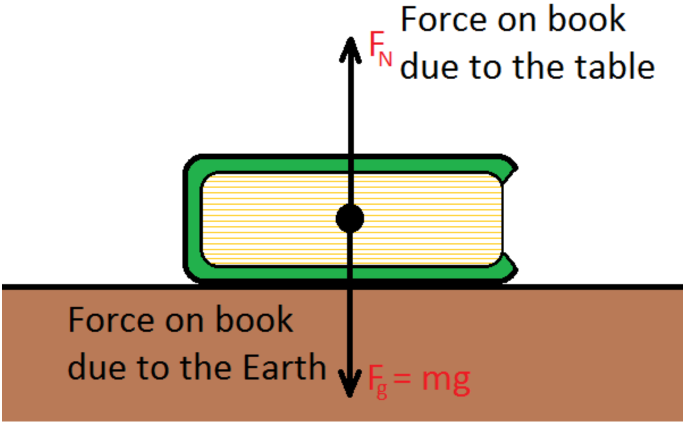

Are these two forces a Third Law 'action-reaction' pair?
(A) Yes, they are equal and opposite. (B) No. (If so, tell me why!)

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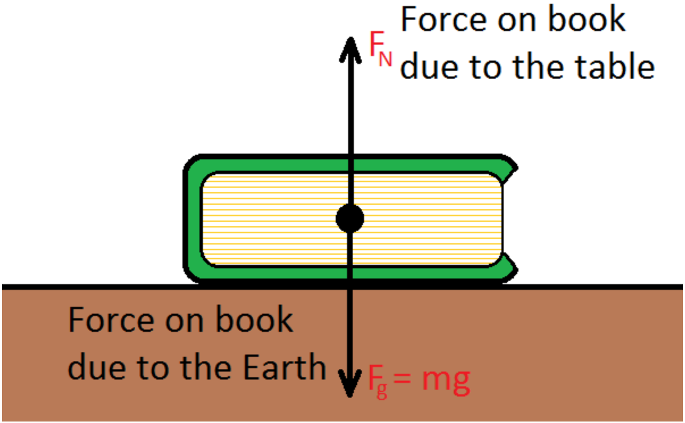

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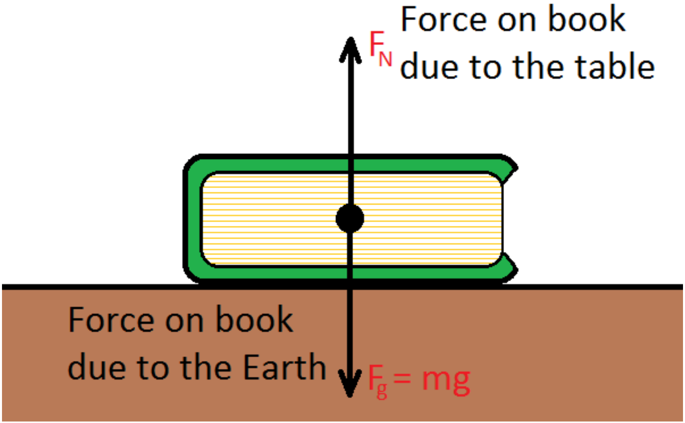
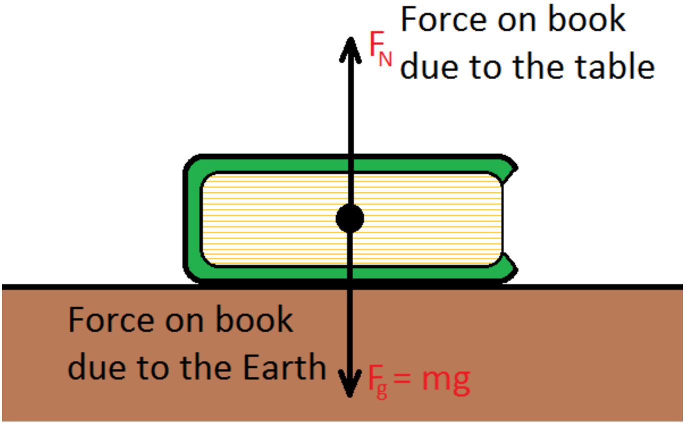


No! An action-reaction pair of forces have to be between the same two objects, and if the action acts on one object of the pair, the reaction has to act on the other object of the pair.



So what are the reactions to these forces acting on the book?

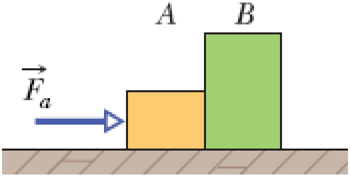
Problem

Another problem.

Problem

In figure (a), a constant horizontal force $\vec{F}_a$ is applied to block A, which pushes against block B with a 20.0 N force directed horizontally to the right. In figure (b), the same force $\vec{F}_a$ is applied to block B; now block A pushes on block B with a 10.0 N force directed horizontally to the left. The blocks have a combined mass of 12.0 kg. What are the magnitudes of

1. their acceleration in figure (a), and
2. the force $\vec{F}_a$?



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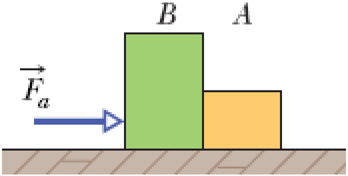

(b)

Newton's Third Law—where does it come from?

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When we say that momentum is conserved, what we mean is:

$$\vec{P}_{total;initial} = \vec{P}_{total;final}$$

which means $\vec{P}_{total}$ stays constant throughout - thus it is conserved.

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$$\Delta\vec{p}_{12} + \Delta\vec{p}_{21} = 0$$

where $\Delta\vec{p}_{12}$ is the change in $\vec{p}_1$ due to object 2, and $\Delta\vec{p}_{21}$ is the change in $\vec{p}_2$ due to object 1.

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where $\Delta\vec{p}_{12}$ is the change in $\vec{p}_1$ due to object 2, and $\Delta\vec{p}_{21}$ is the change in $\vec{p}_2$ due to object 1. This interaction takes place over a period of time Δt, so we can write:

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This is Newton's Third Law, and it is a consequence of the Law of Conservation of Momentum.

General Note

For massless ropes, the net force acting on them must be zero, so in general, the tension at one end of the rope will be equal to the tension at the other. If we include pulleys, we must check if the pulley has friction or mass. If it does, the tension after passing over the pulley may be different. If the pulley is both massless and frictionless, the tension should not change after passing over the pulley.