AAPT PHYSICS EDUCATION
AMERICAN JOURNAL of PHYSICS

Do advanced physics students learn from their mistakes without explicit intervention?

Andrew Mason and Chandralekha Singh

Citation: Am. J. Phys. **78**, 760 (2010); doi: 10.1119/1.3318805
View online: http://dx.doi.org/10.1119/1.3318805
View Table of Contents: http://ajp.aapt.org/resource/1/AJPIAS/v78/i7
Published by the American Association of Physics Teachers

Related Articles

Discipline-based education research: Understanding and Improving Learning in Undergraduate Science and Engineering, an NAP/NSF published book on DBER, tinyurl.com/WS-DBER
Phys. Teach. 50, 510 (2012)

Preparing tutorial and recitation instructors: A pedagogical approach to focusing attention on content and student reasoning
Am. J. Phys. 80, 1020 (2012)

Thinking like a physicist: A multi-semester case study of junior-level electricity and magnetism
Am. J. Phys. 80, 923 (2012)

An item response curves analysis of the Force Concept Inventory
Am. J. Phys. 80, 825 (2012)

Chronicling a successful secondary implementation of Studio Physics
Am. J. Phys. 80, 832 (2012)

Additional information on Am. J. Phys.

Journal Homepage: http://ajp.aapt.org/
Journal Information: http://ajp.aapt.org/about/about_the_journal
Top downloads: http://ajp.aapt.org/most_downloaded
Information for Authors: http://ajp.dickinson.edu/Contributors/contGenInfo.html

ADVERTISEMENT

WebAssign.
The PREFERRED Online Homework Solution for Physics
Every textbook publisher agrees! Whichever physics text you're using, we have the proven online homework solution you need. WebAssign supports every major physics textbook from every major publisher.
webassign.net

Downloaded 26 Oct 2012 to 128.118.113.136. Redistribution subject to AAPT license or copyright; see http://ajp.aapt.org/authors/copyright_permission

Do advanced physics students learn from their mistakes without explicit intervention?

Andrew Mason and Chandralekha Singh
Department of Physics and Astronomy, University of Pittsburgh, Pittsburgh, Pennsylvania 15260

(Received 2 March 2009; accepted 25 January 2010)

We discuss a case study in which 14 advanced undergraduate physics students taking an honor-level quantum mechanics course were given the same four problems on midterm and final exams. The solutions to the midterm problems were provided to students. Their performance on the final exam shows that although some advanced students performed equally well or improved compared to their performance on the midterm exam on the problems given twice, a comparable number performed less well on the final than on the midterm exam. The wide distribution of students' performance on problems given again suggests that most advanced students do not automatically use their mistakes as an opportunity for learning, repairing, extending, and organizing their knowledge structure. Interviews with a subset of the students revealed attitudes toward problem solving and gave insight into their approach to learning.
[DOI: 10.1119/1.3318805]

I. INTRODUCTION

Helping students learn to think like physicists is a major goal of most physics courses from the introductory to the advanced.[1–6] Expert physicists monitor their own learning and use problem solving as an opportunity for learning, extending, and organizing their knowledge.[7,8] Prior work has focused on how introductory physics students differ from physics experts[9–13] and strategies that may help introductory students learn to learn.[14–18] Few investigations have focused on the learning skills of advanced physics students, although some investigations have been done on the difficulties that advanced students have with various advanced topics.[19–23]

It is commonly assumed that most students who have completed an undergraduate physics curriculum have not only learned a wide body of physics content but have also picked up the habits of mind and self-monitoring skills needed to build a robust knowledge structure.[9] Instructors take for granted that advanced physics students will learn from their own mistakes in problem solving without explicit prompting, especially if students are given access to clear solutions. It is implicitly assumed that unlike introductory students, advanced students have become independent learners and will take the time to learn from their mistakes even if the instructors do not reward them for fixing them, for example, by explicitly asking them to turn in, for course credit, a summary of the mistakes they made and writing how those mistakes can be corrected.[24–28]

Little is actually known about whether the development of these skills from the introductory level until the students become physics professors is a continuous process of development or whether there are some discontinuous "boosts" in this process, for example, when they become involved in graduate research or when they independently start teaching and researching. There is also no research data on the fraction of students who have gone through the traditional physics curriculum and have been unable to develop sufficient learning and self-monitoring skills.

Investigations in which advanced physics students are asked to perform tasks related to simple introductory physics content do not properly assess their learning and self-monitoring skills.[9,10] Advanced students may possess a large amount of "compiled knowledge" about introductory physics and may not need to do much self-monitoring or learning while dealing with introductory problems.

The task of evaluating advanced physics students' learning and self-monitoring skills should involve advanced-level physics topics at the periphery of advanced students' own understanding. Although tracking the same student's learning and self-monitoring skills longitudinally is a difficult task, taking snapshots of advanced students' learning and self-monitoring skills can be very valuable.

In this paper we investigate whether students in an advanced quantum mechanics course learn from their own mistakes without explicit intervention. Honor-level quantum mechanics at the University of Pittsburgh is a two-semester course sequence that is required only for those students who want to obtain an honor degree in physics. It is often one of the last courses an undergraduate physics major takes. We administered four quantum physics problems in the same semester both on the midterm and final exams. Solutions to all of the midterm questions were available to students on the course website. Written feedback was provided to students after their midterm performance, indicating on the exams where mistakes were made and how they could be corrected.

Our goal was to explore the extent to which these advanced physics students use their mistakes as a learning opportunity[29] and whether their performance on problems administered on the final exam was significantly better than the performance on those problems in the midterm exams. We also interviewed a subset of students individually using a think-aloud protocol[30–32] within 2 months to obtain a deeper understanding of students' attitudes and approaches to problem solving and learning.[33,34] Evaluating how well the students were able to retrieve relevant knowledge to solve the quantum mechanics problems during the interviews gave a glimpse of the robustness of students' knowledge structure.[35]

II. PROCEDURE

The honor-level quantum mechanics course had 14 students; most were physics majors in their senior year. The class was primarily taught in a traditional lecture format, but the instructor (one of the authors) had the students work on several preliminary tutorials that were being developed. Students were assigned weekly homework during the 15-week semester. There were two midterm exams and a comprehensive final. The midterms covered only limited topics. Students had instruction in all relevant concepts before the exams, and homework was assigned each week from the material covered in that week. Each week, the instructor held an optional class in which students could ask for help on any relevant material in addition to holding office hours. The first midterm took place approximately 8 weeks after the semester started, and the second midterm took place 4 weeks after the first midterm. We selected two problems from each of the midterms and gave them again verbatim on the final exam (see the Appendix) along with other problems not asked earlier.

Problem 1 on expectation values, Problem 2 on measurements, and Problem 3 on momentum were ones with which several students had difficulty; Problem 4 on the harmonic oscillator, which most students found straightforward, was also chosen. The most difficult of the problems was Problem 3, which was also assigned as a homework problem before the midterm exam but was thought by students to be more abstract than the other problems. Problem 4 was solved in the assigned textbook.[37] The students had access to the homework solutions and midterm problems. Thus, students had the opportunity to learn from their mistakes before they encountered the four problems on the final.

A scoring rubric, developed jointly with Yerushalmi and Cohen[24–28] to assess how well the students in introductory physics courses diagnose their mistakes when explicitly prompted to do so, was adapted to score students' performance on the four problems. The scoring was checked independently by another scorer, and at least 80% agreement was found on the scoring for each student on each problem in each attempt.

One section of the scoring rubric scores students on their physics performance, and the other sections score how well they presented their solution. The rubric for the presentation part was somewhat different from the corresponding part for introductory physics[24–28] because quantum mechanics problems often ask for more abstract answers (for example, showing that certain energy eigenstates are equally probable) in contrast to finding a numerical answer. Therefore, some categories in the introductory physics rubric (for example, giving units) were omitted from the presentation part of the quantum mechanics rubric, and other categories were adapted to reflect the nature of the quantum problems better (for example, checking that the answer was adapted to making a conceptual connection with the results).

In-depth interviews lasting 1–1.5 h were conducted with four paid student volunteers from the group of 14 students in the following semester within the first 2 months using a think-aloud protocol.[30–32] At that time three of the four interviewed students were enrolled in the second semester course in honor-level quantum mechanics. The fourth student had graduated in the Fall semester and was performing research with a faculty member. During these interviews, we first asked students about their approaches and strategies for problem solving and learning and asked them to solve the same four problems again while thinking aloud. We did not disturb them initially when they answered the questions and asked only for clarification of points after the student had answered the questions to the best of his/her ability. These interviews also provided an opportunity to understand how well students had retained relevant knowledge and could retrieve it a couple of months later to solve the problems. Two shorter interviews were conducted later with two additional students and mainly focused on students' attitudes and approaches to learning due to the time constraints.

III. RUBRICS AND SCORING

Table I demonstrates the scoring rubric for Problem 3 along with the score of student 6 on the midterm and final exams. In the following we describe the symbols used for scoring and then explain how a quantitative score is derived after the initial scoring is assigned symbolically for each subpart of the rubric. The symbol "+" (1 point) is assigned if a student correctly completes a task as defined by the criterion for a given row. The symbol "−" (0 points) is assigned if the student either fails to do the given task or does it incorrectly. If a student is judged to have given answer that was partially correct, the rater may assign a combination of pluses and minuses $(++/-,+/-,+/--)$, with the understanding that such a combination represents an average score of pluses and minuses (for example, $++/-$ is equivalent to 2/3 of a point). If a student's solution does not address a criterion, then "n/a" (not applicable) is assigned, and the criterion is not considered for grading purposes. For example, if the student does not invoke a principle, the student will receive a − in the invoking row but will receive n/a for applying it in the apply row because the student is not expected to apply a principle that he/she did not invoke.

An overall or cumulative score was tabulated for each of the physics and presentation parts for each question. The cumulative score for the presentation or problem solving part was calculated by averaging over the scores for each of the subcategories (organization, plan, and evaluation) in each column for each student on a given problem on the midterm or final exams.

IV. RESULTS

Although the grading rubric allows us to assign scores to each student for performance on physics and presentation parts separately, these two scores are highly correlated with the regression coefficient between the two scores equal to $R=0.98$. The reason for this high correlation is that the students' presentation of the problem depended on whether or not they understood the physical content. If a student did not know the relevant physical concepts, he/she could not set up an appropriate problem solving strategy to score well on the presentation part. We therefore only focus on students' physics scores on each of the four problems. The physics scores for each student for each of the four problems were analyzed as separate data points for a total of 56 data points. A separate analysis that omitted the harmonic oscillator problem was also done to focus on the three difficult problems for a total of 42 data points.

The average midterm score of all students was 66% on the four problems and 57% on the three difficult problems. The average final exam score of all students was 60% on the four problems and 53% on the three difficult problems. Thus, the students' average final exam performance on these problems is slightly worse than their performance on the midterm exams. This decrease of the average score in the final exam

Table I. Sample scoring of student 6 in midterm and final exams on the momentum problem.

		Sample student scores	
General categories	Specific criteria	Midterm diagnosis of solution	Final exam diagnosis of solution
Invoking appropriate concepts	(1) Taylor expansion definition: $f(x+x_0)=\sum_{n=0}^{\infty}\frac{1}{n!}x_0^n\left(\frac{d}{dx}\right)^n f(x)$	+	−
	(2) Definition of momentum operator in position space in one dimension: $\hat{p}=-i\hbar(d/dx)$	+	+
	(3) Expansion of exponential: $e^u=\sum_{n=0}^{\infty}\frac{1}{n!}u^n$	+	+
Invoking inappropriate concepts	(4) Valid principles or concepts but not relevant (for example, expectation values)	n/a	− (Fourier transform)
	(5) Invalid principles or concepts (for example, confusing position and momentum space)	n/a	− (incorrect reasoning)
Applying concepts	(1) Partial derivative in terms of momentum operator: $\partial/\partial x=i\hat{p}/\hbar$	+	+
	(2) $e^{i\hat{p}x_0/\hbar}=\sum_n\frac{1}{n!}x_0^n\left(\frac{i\hat{p}}{\hbar}\right)^n$	+	++/− (constants incorrect)
	(3) Taylor expansion performed correctly to obtain $f(x+x_0)=e^{i\hat{p}x_0/\hbar}f(x)$	+	−
Organization	Clear/appropriate knowns, for example, $e^u=\sum_{n=0}^{\infty}\frac{1}{n!}u^n$	+	−
Plan	(1) Appropriate target quantity chosen, (2) appropriate intermediate variables chosen, and (3) consistent plan	+	−
Evaluation	(1) Completes proof: $f(x+x_0)=e^{i\hat{p}x_0/\hbar}f(x)$ and (2) makes connection with results (momentum operator is generator of translation in space)	+	+/−
Overall	Physics (%)	100	48
Score	Presentation (%)	100	17

compared to the midterm exams suggests that the assumption that the senior-level physics majors will automatically learn from their mistakes may not be valid.

Table II contains the students' scores on the four problems for both the midterm and final exams. In addition, an average midterm score m_i and a final exam score f_i is given for each student. We see that some students did well on both exams or improved in performance, but others did poorly both times or

Table II. Individual student performances on the four problems and their averages in the midterm and final exams.

	Problem 1 (expectation value)		Problem 2 (measurement)		Problem 3 (momentum)		Problem 4 (harmonic oscillator)		Average	
Student	Mid	Final	Mid	Final	Mid	Final	Mid	Final	Mid (m_i)	Final (f_i)
1	78	100	68	23	0	0	100	100	62	56
2	50	8	100	48	5	0	83	10	60	17
3	100	18	88	88	83	100	100	83	93	72
4	29	31	21	4	83	0	100	100	58	34
5	14	28	38	0	10	29	100	100	41	39
6	100	15	52	23	100	48	100	100	88	47
7	42	58	33	29	0	83	85	83	40	63
8	100	100	100	88	30	100	100	100	83	97
9	100	100	87	87	83	83	100	100	93	93
10	100	63	26	81	3	0	94	42	56	47
11	33	100	88	88	100	100	100	100	80	97
12	100	100	100	100	100	100	100	100	100	100
13	100	94	18	88	0	4	100	71	54	64
14	13	0	27	13	0	0	37	32	19	11

Table III. Total percentage and number of instances in which students who performed above 60% on the midterms continued to perform above that threshold (good to good) or regressed (good to bad) and the number and percentage of instances in which students who performed below 60% on midterms continued to perform below that threshold (bad to bad) or improved (bad to good). The number of instances are shown for all problems together (total instances) and for the four problems separately.

	Good to good	Good to bad	Bad to good	Bad to bad
Problem 1	6	2	1	5
Problem 2	5	2	2	5
Problem 3	4	2	2	6
Problem 4	11	2	0	1
Total instances	26	8	5	17
Total percentage	46%	14%	9%	30%

did worse on the final. Students struggled the most on Problem 3 on both exams and regressed the most from the midterm to final exam on Problem 2.

Table III summarizes the students' performance on the problems. For the sake of comparison, "good" is defined as obtaining 60% or higher using the rubric on the physics score, and "bad" is defined as getting less than 60%. The "good to good" category means that a student performed better than 60% on both exams, and "good to bad" means that a student performed higher than 60% on the midterm and below 60% on the final. For three of the four problems, the results are mixed and are split nearly evenly between students who performed well on both exams and students who performed poorly. The harmonic oscillator problem is the problem that most students had very little trouble with on either attempt.

For the students who regressed from the midterm to the final exam, we observe a pattern where students who got a question right on the midterm employed a different procedure for the same question on the final exam. The procedure used was often a technique learned in the second half of the course and was not relevant to solving the problem. This regression suggests that some students are applying memorized procedures, rather than trying to understand the problem they are solving. This pattern was observed in approximately 14% of the cases.

As shown in Tables II and III, approximately 30% of the students performed poorly on both exams. In many of these cases, students wrote extensively on topics that were irrelevant to the question. It is difficult to imagine that the students did not know that what they wrote was not relevant to the questions. It is possible that the students thought that if they wrote anything that they could remember about the topic (whether relevant or not), they might get some points for trying. Often, the irrelevant writings of a student on a particular question were different on the midterm and final exams. The poor performance of students both times suggests that when the midterm exam was returned to them and the correct solution was provided, they did not use their mistakes as an opportunity for learning.

Examples of midterm and final exam answers of students who performed poorly both times and solved a problem correctly in the midterm exam but regressed in the final exam or improved a second time are available,[36] as well the solution key.

V. INTERVIEWS

The regression of some students from the midterm to final exam is contrary to the belief that physics majors who have almost graduated have already learned to learn and will take the opportunity to learn from their mistakes. One hypothesis is that students who regress on problems from the midterm to the final exam might have memorized how to do certain problems (especially because the material for each of the midterm exams was not extensive) rather than learning the concepts, organizing and extending their knowledge structure, and ensuring that the new knowledge learned via homework and other course material is integrated with their prior knowledge.

We cannot obtain the whole picture of what a student is thinking by examining the final written product of their work. We asked the class for paid volunteers for individual interviews lasting between 60 and 90 min each). Four of the original 14 students responded. The interviews took place about 2 months after the end of the quantum mechanics class in which the written tests were given, and with the exception of one student (student 10), all subjects were currently enrolled in the second semester course in quantum mechanics. The timing for the interviews was chosen because it ensured that the students could not simply reproduce answers they may have memorized before exams, and it allowed us to test long-term learning and knowledge organization of students by determining how well the students could do these problems several months after the semester was over.

In addition to these four interviews, two other students (students 8 and 9) were later available for shorter interviews. These last two interviews focused only on their attitudes and approaches toward problem solving and learning. They were not asked to solve the problems again during the interview due to time constraints.

Each of the interviewed students was first asked a series of questions that sought the student's opinion on various aspects of the course and on the student's performance in the course. Students were also asked about their attitudes and approaches to problem solving and learning. In particular, they were asked about their general approach to problem solving and learning in physics, whether they prefer to do homework alone or with peers, whether they take the time to learn from their mistakes on homework and exams as soon as possible, and their performance on each question, for example, why they struggled with one problem or did well on another. The interviews corroborate the fact that similar to introductory physics students, many advanced students need explicit guidance and support in exploiting problem solving as an opportunity for learning and in reflecting and learning from their mistakes and in building a robust knowledge structure.

Each of the first four students interviewed was then presented with the four problems to be solved one by one. For each problem, the student first attempted to solve the problem without any input from the interviewer using a think-aloud protocol. If a student noted that he did not know how to solve the problem correctly, the researcher first encouraged him to progress as far as he could. Then, the researcher provided successive hints to allow the student to go back, fix mistakes, and make more progress. Using this process, we got insight into how well a student could self-repair on the spot[29] and solve the problem correctly with the scaffolding provided by the researcher.

The four students who volunteered for the interviews were students 3, 6, 10, and 11. Student 3 did very well on the midterm exam but had some trouble on the final exam (he regressed on Problem 1). Because of this regression, it was interesting to examine this student's problem solving and learning approach and explore how well he retained and could retrieve the knowledge acquired the previous semester. Student 6 generally did well on the problems on the midterm exams but performed poorly on three problems on the final.

According to his performance on the midterm and final exams, student 10 was a weak student. He did well on Problem 1 both times, improved from a poor performance on Problem 2, regressed from a good performance on Problem 4, and fared poorly on both attempts on Problem 3. Student 10 graduated immediately after taking the course, and he was doing research with a faculty member in the department. It was interesting to see how well he had retained and could retrieve relevant knowledge while solving problems based on last semester's material in comparison to the three students who were in the second semester quantum mechanics course (although the material for the second semester course is mostly unrelated to the first semester material).

Student 11 did well on the midterm except for Problem 1, and he improved on the final. We wanted to understand how he learned from his mistakes and wanted to explore how well he had retained the knowledge beyond the final exam.

The shorter interviews conducted later about attitudes and approaches to problem solving involved students 8 and 9 who had performed well in the final exam.

The interviewed students had varied study habits. For example, some liked to work with other students, while others preferred to work alone. Student 3 preferred working alone except when he found a problem tricky, in which case he would consult the professor or occasionally a classmate who understood the problem better. The reason he worked alone was that he found it too much trouble to get a regular study group together. His study habits for exams focused mostly on doing practice problems and reading the book for concepts, saying that these study approaches were about all that could be done to study for exams because problems were the main focus of study. He explicitly noted that he looked at the homework and exam solutions right before the exam.

Student 6 stated that his study habits were generally consistent throughout the semester. He cited working with student 10 on the homework, specifically comparing notes with each other after each person worked individually on the homework and that he made sure to speak with the instructor for any mistakes made on the homework or exams. Furthermore, he said that he created a study guide and studied equations as well as reviewed with student 10 for 1 week before each exam. His opinion of the class was very high—the quantum mechanics classes were his favorite of the entire physics curriculum—and he felt that the class, while difficult, could be completed successfully with effort.

Student 10 stated that his performance in the quantum mechanics class was his weakest performance of his Fall semester classes. He found the course more difficult than usual because he considered the material unique with regard to the other senior-level course material. His study habits for the midterms seemed fairly structured: First he would read the text material relevant to the exam; next, he created a study sheet by writing down key sample problems and equations (he did not study the equations *per se*, but the act of copying down key sample problems and equations helped him to study). Finally, he redid as many homework problems as possible. He indicated that a poor performance on homework and exams would make him prefer to put away the assignment without looking it over. He also stated that if a problem occurred on an exam that he had seen before in the homework, he would probably do better the second time around.

Student 11 suggested that his study habits were shaped by the available sources. He especially credited the course textbook[37] for providing examples and conceptual questions as well as homework problems that helped him learn the course material. He also credited homework help sessions given in the class during which any gaps in the textbook's treatment of a problem would be filled in. He worked alone on homework during the course, only interacting with other students if he got stuck on a problem, and would put it off until the last moment but would check his mistakes on the homework as a "matter of principle" (for as much effort as he put into the homework, if he got a problem wrong anyway, he wanted to know why). He also consulted the internet to find hints to problems similar to the homework. This student was also a mathematics major as well as a physics major, and he stated several times that he felt a math method course helped him on the material.

Discussions with students about their attitudes and approaches to problem solving suggest that they often study based on what they expect to see on an exam rather than learning from their mistakes and building a robust knowledge structure. For example, student 10 stated that he struggled on a problem that first appeared on a midterm exam and then on the final because he did not expect a problem to be repeated on the final exam and did not practice them well. Student 8 described a similar approach: "If I make mistakes in the homework, I look at the TA's solutions carefully because I know those problems can show up in the exams. But if I make a mistake in the midterm exam, I won't be so concerned about what I did wrong because I don't expect those questions to show up in the final exam. Also, if I don't do well on the exam, I don't feel like finding out what I did wrong because reading my mistake again would just hurt me again, and I don't want anything to ruin the after-exam happy time."

Student 9 claimed he would look back to see what mistakes he made but would be more careful doing this for mistakes in the homework than in the exams.

Similarly, when asked to solve Problem 3, all four students struggled to some extent, ranging from forgetting or misunderstanding details about the Taylor expansion to having no idea how to even start the problem. All four students complained about Problem 3, stating that they did not study it as thoroughly for exams because they didn't think they would be tested on it. For example, student 3 claimed that the required proof didn't seem very physical: "…you know, sometimes, really mathematical problems… I mean, this isn't terribly mathematical but sometimes problems like this just seem like 'oh, this is just a math thing.' You know [the book] or even the professor, maybe prior to the problem doesn't tell you the importance of the problem, like what it really demonstrates. I remember [the professor] did [afterwards].… But [the book] doesn't say anything, it just says, 'Do it.' I remember thinking, at the time I thought it was just something to make me stay up another hour and a half. And then, you know, it was on the test and I thought, 'I should've paid more attention to it,' and then it was on the final and I thought, 'jeez, I really should've paid attention to it.'"

Similarly, student 6 claimed that Problem 3 did not stand out as something important from the examination point of view. After writing an expression for the momentum operator he added "…yeah, I totally forget… I haven't touched translator of space since the final… I remember the translator in time and that goes the same way. I remember you got to use some dummy function, some function '*f*' that you can throw in, and that you can sum it…. I remember there was an exponential… it's just not ringing a bell right now."

Student 10 also noted that he did not think that Problem 3 was important: "It was just one of the problems in the homework. It was never mentioned previously or after, so I didn't assign much importance to it in my head as far as studying goes to it." This type of selective studying based on the probability of the material being on the exam suggests that even advanced upper-level students attempt to game/play the system by anticipating what problems will be on the exams rather than trying to integrate new knowledge with their prior knowledge to build a robust knowledge structure.

Students 3, 6, and 10 had difficulties with Problem 1. In this problem, they struggled to remember how to use Dirac notation, stating that they had a difficult time mastering it in the course and the second semester quantum mechanics course hardly ever used this notation. Student 3 had done Problem 1 involving Dirac notation correctly on the midterm exam but did not know how to do it on the final exam. The same student continued "I remember doing it, I remember seeing it, I remember thinking, 'oh, it's not really that bad,' so I know it's not bad. It's just… I'm not remembering how to manipulate with bra and ket. Basically that's all it is." Then, he added: "…yeah, I mean that's really what the killer thing is… yeah, I mean it's just the bra and ket notation really."

Student 3 appears to have memorized some procedures involving Dirac notation, which he used to perform well on Problem 1 in the midterm. However, he did not remember how to do the problem during the final exam and the interview because his knowledge about Dirac notation was not properly integrated into his knowledge structure. It may be easier to discern if students have a robust knowledge structure when the amount of material on which they are evaluated is not very limited because limited material may make invoking of relevant concepts easy by memorizing a few procedures, and all that may be required to perform well is the knowledge of how to apply the limited number of memorized procedures to solve a problem even if a conceptual understanding is lacking.

After struggling with Problem 1, student 10 stated that he felt that the transition from integral notation (position representation) to Dirac notation was not clear to him. It is possible that Dirac notation could be construed as unintuitive to students learning it for the first time. Alternately, the regression on the final from the midterm by students 3 and 6 suggests they may have crammed on Dirac notation to "get by" on the midterm even when they did not understand it well.

For Problem 2 students 3, 6, and 10 had a difficult time figuring out what they were supposed to do to solve it. Student 3 was unclear on how to get started and required a hint from the interviewer to recall how to do it correctly. Student 6 displayed common errors such as confusing energy eigenstates with position eigenstates and said that he failed to learn from mistakes on his midterm exam attempt because his solution procedure, while incorrect, gave an answer that seemed correct, that is, that all even energy eigenstates are equally probable (actually, the probability of collapsing into an even energy eigenstate after a measurement of energy is zero if the initial state is a delta function in the middle of the well because there is no overlap between the initial delta function wave function and the even energy eigenstate wave functions that have a node in the middle of the well). Student 10 tried to recall some relevant concepts but did not remember the necessary tools to solve the problem. For example, while attempting Problem 2, student 10 noted: "…just from my memory, like, there's just too many holes and stuff because I haven't looked at it or thought about it in a while…" He also had similar issues with the other problems. For example, after struggling with the harmonic oscillator problem, student 10 noted "I feel like I might just be taking it in a different, in the wrong direction from that point, but… basically, yeah. Like where it wouldn't take much to get me to remember how to do this again completely."

In contrast, student 11 displayed excellent physical and mathematical understanding of the problems during the interview. Unlike the other three students, he solved all but Problem 3 during the interview without getting stuck on anything and only needed a small hint to complete Problem 3. While answering Problem 1 during the interview, student 11 noted, "Well, we learned that in particular. It was proven to us in, like, three different ways. I remember the page in Griffiths now…" He cited that the reason for his success was that the course material and study resources were very good and that he was able to learn from it and develop a good understanding of quantum mechanics. He explicitly said that he used his mistakes in problem solving for reflecting about the holes in his understanding and for overcoming those difficulties. In addition, he majored in mathematics as well as in physics and said that the math course he took, for example, linear algebra, helped him very much in understanding the concepts in quantum mechanics.

VI. SUMMARY AND CONCLUSIONS

Prior research on the problem solving and self-monitoring skills of introductory physics students demonstrates that the introductory students do not learn these skills automatically, for example, by listening passively to lectures and having access to solved examples.[24–28] Many introductory physics students are "captive audiences"—they might not buy into the goals of the course, and their main goal becomes getting a good grade even if their learning is superficial.[16] Research shows that these introductory physics students can benefit from explicit guidance and feedback in developing problem solving and learning skills and alignment of course goals with assessment methods.[14–18,24–28,38,39]

We found that advanced students in the honor-level quantum mechanics sequence did not automatically improve their performance on identical questions given in midterm and final exams. The students were provided the correct solutions and their own graded exams. Even then, there was an apparent lack of reflective practice, and many students did not take the opportunity to repair and organize their knowledge structure.

We probed students' attitudes and approaches toward problem solving and learning and asked them to solve the same problems again. The results were consistent with students' "self-described" approaches toward problem solving and learning. We also found evidence that even in these advanced courses, there are students who do not use their mistakes as an opportunity for learning and for building a robust knowledge structure; they resort to rote learning strategies for getting through the course. One interviewed student alluded to the fact that he always looked at the provided homework solutions but did not always look up the correct midterm exam solutions partly because he did not expect these questions to be repeated on the final exam.

Individual discussions with some physics faculty members suggests that sometimes their incorrect inferences about advanced physics students' learning and self-monitoring skills are based on their assumption that all physics majors are like them. They may not appreciate the large diversity in the population of physics majors and may not realize that those who become college physics faculty consist of a very select group of physics majors. Although longitudinal research is needed to investigate the differences between those advanced students who become physics faculty and those who do not, it is possible that those students aspiring to be physics faculty make more effort to learn from their own mistakes.

Similar to introductory physics students, advanced physics students may benefit from explicit scaffolding support and guidance to help them become independent learners. Students will automatically use problem solving as an opportunity for reflecting and learning if they are intrinsically motivated to learn the content and to extend and organize their knowledge.[40–43] However, students who are not intrinsically motivated may need extrinsic motivation, for example, explicit reward for developing higher order thinking and self-monitoring skills. Instructional strategies that aim to achieve these goals must ensure that the instructional design and method of assessment are aligned with these goals in order for the students to take them seriously.

There are a number of strategies based on formative assessment that can provide explicit guidance and extrinsic motivation to learn. These instructional strategies not only show students where they need to improve but also provide them with a path or opportunity to improve. For example, Etkina *et al.*[38,39] documented that introductory physics students can be taught the process of science with explicit intervention that provides them with scaffolding support, which lasts at least for 6 weeks. A reward system (for example, grade incentive) is critical to help students learn to self-monitor their work. One strategy is explicitly asking students to fix their mistakes by circling what they did incorrectly in homework assignments, quizzes, and exams and explaining why it is incorrect and how it can be done correctly. Asking students to develop "concept maps"[14] after each unit and providing feedback as they learn to connect different concepts can be a useful strategy for helping them develop a robust knowledge structure that will reduce the probability of forgetting concepts. Explicitly asking students to explain in words why a certain principle or concept is relevant to solving a problem and coupling conceptual and quantitative problem solving may be an effective means to force students to reflect on what they are doing and to help them build a more robust knowledge structure.[14–18,24–28] In exploiting each of these strategies to help advanced students learn to learn, assessment should be commensurate with the goals.

ACKNOWLEDGMENTS

The authors are grateful to F. Reif, R. Glaser, E. Yerushalmi, E. Cohen, J. Levy, and R. P. Devaty for useful discussions. The authors thank the National Science Foundation for financial support. The authors thank the referees for constructive feedback.

APPENDIX: THE FOUR PROBLEMS

The following four problems were given both on the midterm and final exams. The problem numbers refer to the numbering of the problems in the final exam. Students were given an additional sheet on which useful information was provided. For example, they were given the stationary state wave functions and energies for a one-dimensional infinite square well. For the one-dimensional harmonic oscillator, they were given the energies in terms of quantum number n, how the ladder operators relate to the position and momentum operators, the commutation relation between the raising and lowering operators, and how a raising or lowering operator acting on the nth energy eigenstate of a one-dimensional harmonic oscillator changes that state.

- Problem 1. The eigenvalue equation for an operator $\hat{Q}$ is given by $\hat{Q}|\psi_i\rangle=\lambda_i|\psi_i\rangle$, $i=1,\ldots,N$. Find an expression for $\langle\psi|\hat{Q}|\psi\rangle$, where $|\psi\rangle$ is a general state, in terms of $\langle\psi_i|\psi\rangle$.
- Problem 2. For an electron in a one-dimensional infinite square well with well boundaries at $x=0$ and $x=a$, the measurement of position yields the value $x=a/2$. Write down the wave function immediately after the position measurement, and without normalizing it show that if energy is measured immediately after the position measurement, it is equally probable to find the electron in any odd-energy stationary state.
- Problem 3. Write an expression to show that the momentum operator $\hat{P}$ is the generator of translation in space. Then prove the relation. (Simply writing the expression is not sufficient…you need to prove it.)
- Problem 4. Find the expectation value of potential energy in the nth energy eigenstate of a one-dimensional harmonic oscillator using the ladder operator method.

[1] A. Van Heuvelen, "Learning to think like a physicist: A review of research based instructional strategies," Am. J. Phys. **59**, 891–897 (1991).

[2] F. Reif, "Scientific approaches to science education," Phys. Today **39** (11), 48–54 (1986).

[3] F. Reif, "Teaching problem solving—A scientific approach," Phys. Teach. **19**, 310–316 (1981).

[4] J. Larkin and F. Reif, "Understanding and teaching problem solving in physics," Eur. J. Sci. Educ. **1** (2), 191–203 (1979).

[5] F. Reif, "Millikan Lecture 1994: Understanding and teaching important scientific thought processes," Am. J. Phys. **63**, 17–32 (1995).

[6] J. Larkin, "Cognition of learning physics," Am. J. Phys. **49** (6), 534–541 (1981).

[7] D. Maloney, in *Handbook of Research on the Teaching and Learning of Science*, edited by D. Gabel (MacMillan, New York, 1994).

[8] J. Heller and F. Reif, "Prescribing effective human problem-solving processes: Problem description in physics," Cogn. Instruct. **1**, 177–216 (1984).

[9] M. T. H. Chi, P. J. Feltovich, and R. Glaser, "Categorization and representation of physics knowledge by experts and novices," Cogn. Sci. **5**, 121–152 (1981).

[10] J. Larkin, J. McDermott, D. Simon, and H. Simon, "Expert and novice performance in solving physics problems," Science **208**, 1335–1342

(1980).

[11] C. Singh, "When physical intuition fails," Am. J. Phys. **70** (11), 1103–1109 (2002).

[12] R. Dufresne, J. Mestre, T. Thaden-Koch, W. Gerace, and W. Leonard, in *Transfer of Learning From a Modern Multidisciplinary Perspective*, edited by J. P. Mestre (Information Age, Greenwich, CT, 2005), pp. 155–215.

[13] P. Hardiman, R. Dufresne, and J. Mestre, "The relation between problem categorization and problem solving among novices and experts," Mem. Cognit. **17**, 627–638 (1989).

[14] B. Eylon and F. Reif, "Effects of knowledge organization on task performance," Cogn. Instruct. **1** (1), 5–44 (1984).

[15] A. Van Heuvelen, "Overview case study physics," Am. J. Phys. **59**, 898–907 (1991).

[16] A. Elby, "Helping physics students learn how to learn," Am. J. Phys. **69**, S54–S64 (2001).

[17] David E. Meltzer, "Relation between students' problem-solving performance and representational format," Am. J. Phys. **73**, 463–478 (2005).

[18] D. J. Ozimek, P. V. Engelhardt, A. G. Bennett, and N. S. Rebello, "Retention and transfer from trigonometry to physics," AIP Conf. Proc. **790**, 173–176 (2004).

[19] C. Singh, "Student understanding of quantum mechanics," Am. J. Phys. **69** (8), 885–896 (2001).

[20] L. Bao and E. Redish, "Understanding probabilistic interpretations of physical systems: A prerequisite to learning quantum physics," Am. J. Phys. **70** (3), 210–217 (2002).

[21] M. Wittmann, R. Steinberg, and E. Redish, "Investigating student understanding of quantum physics: Spontaneous models of conductivity," Am. J. Phys. **70** (3), 218–226 (2002).

[22] C. Singh, M. Belloni, and W. Christian, "Improving student's understanding of quantum mechanics," Phys. Today **59** (8), 43–49 (2006).

[23] C. Singh, "Student understanding of quantum mechanics at the beginning of graduate instruction," Am. J. Phys. **76** (3), 277–287 (2008).

[24] A. Mason, E. Cohen, E. Yerushalmi, and C. Singh, "Identifying differences in diagnostic skills between physics students: Developing a rubric," in *Proceedings of the Phys. Educ. Res. Conference*, Edmonton, CA, edited by L. Hsu, C. Henderson, and M. Sabella (AIP, Melville, 2008) [AIP Conf. Proc. **1064**, 147–150 (2008)].

[25] E. Cohen, A. Mason, C. Singh, and E. Yerushalmi, "Identifying differences in diagnostic skills between physics students: Students' self-diagnostic performance given alternative scaffolding," *Proceedings of the Physics Education Research Conference*, Edmonton, CA, edited by L. Hsu, C. Henderson, and M. Sabella (AIP, Melville, 2008) [AIP Conf. Proc. **1064**, 99–102 (2008)].

[26] E. Yerushalmi, A. Mason, E. Cohen, and C. Singh, "Effect of self-diagnosis on subsequent problem solving performance," *Proceedings of the Physics Education Research Conference*, Edmonton, CA, edited by L. Hsu, C. Henderson, and M. Sabella (AIP, Melville, 2008) [AIP Conf. Proc. **1064**, 53–56 (2008)].

[27] E. Yerushalmi, C. Singh, and B. Eylon, "Physics learning in the context of scaffolded diagnostic tasks (I): The experimental setup," *Proceedings of the Physics Education Research Conference*, Greensboro, NC, edited by L. Hsu, L. McCullough, and C. Henderson (AIP, Melville, 2007) [AIP Conf. Proc. **951**, 27–30 (2007)].

[28] C. Singh, E. Yerushalmi, and B. Eylon, "Physics learning in the context of scaffolded diagnostic tasks (II): Preliminary results," *Proceedings of the Physics Education Research Conference*, Greensboro, NC, edited by L. Hsu, L. McCullough, and C. Henderson (AIP, Melville, 2007) [AIP Conf. Proc. **951**, 31–34 (2007).

[29] M. T. H. Chi, M. Bassok, M. W. Lewis, P. Reimann, and R. Glaser, "Self-explanations: How students study and use examples in learning to solve problems," Cogn. Sci. **13**, 145–182 (1989).

[30] M. T. H. Chi, "Thinking aloud," in *The Think Aloud Method: A Practical Guide to Modeling Cognitive Processes*, edited by M. W. Van Someren, Y. F. Barnard, and J. A. C. Sandberg (Academic, London, 1994), Chap. 1.

[31] M. T. H. Chi, "Quantifying qualitative analysis of verbal data: A practical guide," J. Learn. Sci. **6** (3), 271–315 (1997).

[32] K. Ericsson and H. Simon, *Protocol Analysis: Verbal Reports as Data* (MIT, Cambridge, 1993).

[33] K. Cummings, S. Lockwood, and J. Marx, "Attitudes toward problem solving as predictors of student success in classroom," *Proceedings of the 2003 Physics Education Research Conference*, edited by J. Marx, S. Franklin, and K. Cummings (AIP, Melville, 2004) [AIP Conf. Proc. **720**, 133–136 (2004)].

[34] J. Marx and K. Cummings, "What factors really influence shifts in students' attitudes and expectations in an introductory physics course?," *Proceedings of the 2006 Physics Education Research Conference*, Syracuse, NY, edited by L. McCullough, L. Hsu, and P. Heron (AIP, Melville, 2007) [AIP Conf. Proc. **883**, 101–104 (2007)].

[35] G. Francis, J. Adams, and E. Noonan, "Do they stay fixed," Phys. Teach. **36**, 488–490 (1998).

[36] See supplementary material at http://dx.doi.org/10.1119/1.3318805 for examples of midterm and final exam answers of students and the solution key.

[37] D. Griffiths, *Introduction to Quantum Mechanics*, 2nd ed. (Pearson, Prentice-Hall, Upper Saddle River, NJ, 2005).

[38] A. Karelina and E. Etkina, "Acting like a physicist: Student approach study to experimental design," Phys. Rev. ST Phys. Educ. Res. **3**, 020106-1–12 (2007).

[39] E. Etkina, A. Karelina, and M. Ruibal-Villasenor, "How long does it take? A study of student acquisition of scientific abilities," Phys. Rev. ST Phys. Educ. Res. **4**, 020108-1–15 (2008).

[40] D. J. Stipek, *Motivation to Learn: From Theory to Practice* (Allyn and Bacon, Boston, 1993).

[41] C. Ames, "Classroom: Goals, structures, and student motivation," J. Educ. Psychol. **84** (3), 261–271 (1992).

[42] C. Dweck and E. Leggette, "A social-cognitive approach to motivation and personality," Psychol. Rev. **95** (2), 256–273 (1988).

[43] J. L. Meece, in *Self-Regulation of Learning and Performance*, edited by D. H. Schunk and B. J. Zimmerman (Erlbaum, Hillsdale, NJ, 1994).