Quiz 9

August 8, 2012
Time limit: 20 minutes

Name:

Section:

Important

Show your work; i.e. show the steps in your solution in a clear and logical fashion in order to receive full points. Understanding how to solve physics problems is important, but so is being able to present your reasoning clearly.

Useful Formulae

$$\omega = \frac{\mathrm{d}\theta}{\mathrm{d}t} \qquad v_t = \omega r \qquad a_c = \omega^2 r$$

$$\alpha = \frac{\mathrm{d}\omega}{\mathrm{d}t} \qquad a_t = \alpha r$$

$$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \ldots = \sum_i m_i r_i^2$$

Parallel Axes Theorem: $I_{new} = I_{com} + Md^2$

$$\vec{L} = I\vec{\omega} \qquad \vec{\tau} = I\vec{\alpha} \qquad \vec{\tau} = \vec{r} \times \vec{F}$$

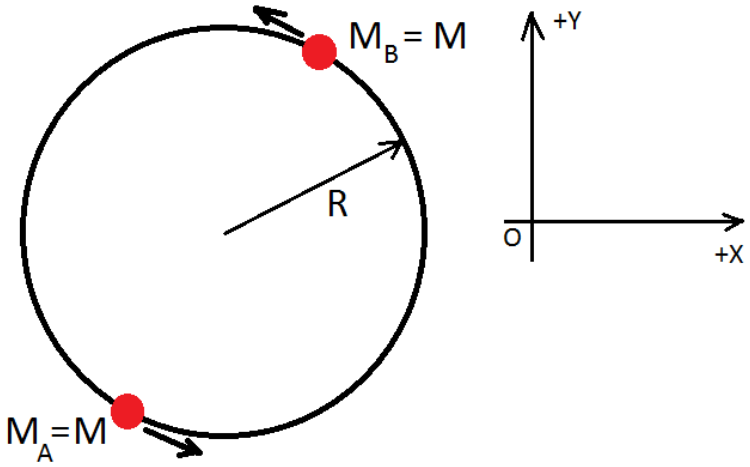
$$\vec{F}_{12} = -G\frac{m_1 m_2 \vec{r}_{12}}{r_{12}^3} = -G\frac{m_1 m_2 \hat{r}_{12}}{r_{12}^2} \qquad U_{12} = -G\frac{m_1 m_2}{r_{12}}$$

$\vec{r}_{12}$ points from m_2 to m_1. $\vec{F}_{12}$ is the force by m_2 on m_1.

Questions

1. In class we looked at several problems where a planet orbited a star. In those problems, we treated the star as stationary and the planet as orbiting in a circle (or ellipse) about the centre of mass of the star. In truth, both the star and the planet orbit around the centre of mass of the *system* (the star plus planet). What usually happens though, is that the star is much more massive than the planet, so that the centre of mass of the system is very close to the centre of mass of just the star, and so the star moves very little, and planet looks like it is orbiting around the centre of the star. But the star still wiggles a little, and astronomers can detect that wiggle, which helps them find planets outside the solar system, orbiting other stars.

   We now look at a more dramatic case: when two stars orbit their common centre of mass. To simplify matters, we will take the masses of the two stars to be equal: $M_A = M_B = M$. The two stars move in a circle of radius R, always on opposite sides, diametrically opposite each other. They are orbiting the centre of mass of the system, which lies at the centre of the circle. Answer the following questions:



(a) What is the distance between the two stars? [0.5 points]

(b) What is the magnitude of the gravitational force of A on B? [2 points]

(c) What is the distance of each star from the centre of mass of the system? [0.5 points]

(d) What force provides the centripetal acceleration of A? [1 points]

(e) In terms of the mass M of each star, Newton's gravitational constant G and the radius R of the orbit, what is the linear speed v of the star B? (They both have the same linear speed, and opposite velocities.) [2 points]

(f) What is the time period of revolution? [2 points]

(g) What is the moment of inertia I_{com} of the system about an axis passing through the centre of mass and perpendicular to the page? [2 points]

(h) In terms of the mass M of each star, Newton's gravitational constant G and the radius R of the orbit, what is the magnitude of the angular momentum L of the system? [6 points]

(i) What is the direction of the angular momentum $\vec{L}$? [3 points]

(j) If the angular momentum of the system stays constant, what can we say about the external torque acting on the system? [1 point]

(k) What is the magnitude of the torque exerted on A by B? (Note that this is an internal torque exerted on A by B) [2 points]

(l) What is the kinetic energy K of the system? [3 points]

(m) What is the potential energy U of the system? [2 points]

(n) What is the total energy of the system? Express your answer in terms of the mass M of each star, Newton's gravitational constant G and the radius R of the orbit. [3 points]

Extra credit If this binary system loses energy by gravitational radiation, so that the total energy decreases, but the masses of the two stars stay the same, what happens to the radius of the orbit? [4 points]

Extra credit What happens to the angular momentum L of the system? [4 points]

i. Increases.
ii. Decreases.
iii. Stays the same.
iv. Stays the same magnitude but changes direction.

Extra credit What is the direction of the torque (if any) acting on the system? If there is no torque, answer: "No direction since there is no torque". [2 points]

Congrats! You just won the Nobel Prize in Physics. See 'Hulse-Tayor Binary' for more details.